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An Improved Glauert Series for Certain Airfoil Problems

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Introduction

In the presence of such complicating effects as unsteadiness, finite span, jet flap, and free-surface interference, for example, an exact solution of the thin airfoil problem is generally out of the question. As a result, it is often convenient to seek a solution for the unknown vortex strength in the form of a truncated Glauert series. In this note we consider the analytical and numerical merits of a slightly modified series solution.

Discussion

Consider, for example, the harmonic motion of an infinitely thin rectangular airfoil of span $2b$ and chord $2a$ in an unbounded inviscid and incompressible stream U. Introducing a right-handed rectangular x', y', z' coordinate system with x' measured downstream, z' normal to the planform, and the origin at the planform centroid, a classical vortex approach leads to the well-known integral equation

$$\bar{w}(x, y) = \frac{1}{4\pi} ⨍_{-A}^{A} ⨍_{-1}^{1} \frac{\partial \bar{\gamma}}{\partial \eta}(\xi, \eta) \times \frac{[(x-\xi)^2 + (y-\eta)^2]^{1/2}}{(x-\xi)(y-\eta)} d\xi d\eta + \frac{ike^{ik}}{2\pi} \bar{\Gamma} \int_1^\infty \frac{e^{-ik\xi}}{\xi - x} d\xi + \bar{D}(x, y) \quad (1)$$

where $\gamma(x, y, t) = \bar{\gamma}(x, y) \exp(i\omega t)$ is the unknown strength per unit chordwise length of the spanwise running bound vortices; $\Gamma(y, t) = \bar{\Gamma}(y) \exp(i\omega t)$ is the resulting circulation; $w(x, y, t) = \bar{w}(x, y) \exp(i\omega t)$ is the upwash given over the airfoil; $D(x, y, t) = \bar{D}(x, y) \exp(i\omega t)$ represents a portion of the downwash terms which is regular over the airfoil and, in particular, at the trailing edge; k is the reduced frequency $\omega a/U$; A is the aspect ratio b/a; $\bar{w}$, $\bar{\gamma}$, and D are nondimensionalized with respect to U, $\bar{\Gamma}$ with respect to Ua, and the coordinates with respect to a. The terms on the right represent the downwash which is induced by the vortex system and which must exactly balance $\bar{w}$.

Typically, the equation is then reduced to a more tractable form by suitable approximation of the square root term. For example, the very high (two-dimensional and lifting line[1] theories) and very low[2,3] aspect ratio theories are equivalent to setting $[(x-\xi)^2 + (y-\eta)^2]^{1/2} \approx |y - \eta|$ and $|x - \xi|$, respectively. Of the various more sophisticated approximations that have appeared in the literature, one of the nicest is due to Laidlaw.[4] He sets

$$[(x-\xi)^2 + (y-\eta)^2]^{1/2} \approx \lambda_0 |x - \xi| + \lambda_\infty |y - \eta| \quad (2)$$

where λ_0 and λ_∞ are functions of aspect ratio and are determined by a least-squares minimization of the difference between the left- and right-hand sides of (2) over the planform. Contemplating the use of a Glauert series of the form

$$\bar{\gamma} = c_0(y) \cot\left(\frac{\phi}{2}\right) + \sum_{n=1}^{\infty} c_n(y) \sin n\phi \quad (3)$$

where $x \equiv -\cos\phi$, Laidlaw observes that the right side of (3) vanishes at the trailing edge $\phi = \pi$, whereas the Kutta condition of zero pressure discontinuity demands that $\bar{\gamma} = -ik\bar{\Gamma}$ there. He therefore modifies his Glauert series by adding on $-ik\bar{\Gamma}(y)$. We point out, however, that the latter may be expanded in a Fourier sine series, and, whether we use the modified or unmodified Glauert series, the resulting solutions for $\bar{\gamma}$ will be identical for $\phi < \pi$ although they will equal $-ik\bar{\Gamma}$ and 0, respectively, at $\phi = \pi$. But this discrepancy will not affect the quantities of physical interest so that either series will suffice.

Although the presence of the additional term is immaterial if an exact solution is carried out, it is generally necessary to truncate the summation at $n = N$ and to follow a collocation solution, for example. It is, therefore, of interest to reexamine the potential merits of a more general modified series,

$$\bar{\gamma} = c_0(y) \cot\left(\frac{\phi}{2}\right) + \sum_{n=1}^{N} c_n(y) \sin n\phi + f(\phi, y) \quad (4)$$

in this light. In doing so, let us reformulate the Kutta condition based upon the integral equation; that is, we seek to satisfy the tangent-flow boundary condition at the trailing edge. Observing that, as $x \to 1$, the right side of (1) behaves as

$$\frac{\lambda_\infty}{2\pi} ⨍_{-1}^{1} \frac{\bar{\gamma}(\xi, y)}{x - \xi} d\xi + \frac{ike^{ik}}{2\pi} \bar{\Gamma} \int_1^\infty \frac{e^{-ik\xi}}{\xi - x} d\xi + 0(1) \quad (5)$$

or

$$-\frac{\lambda_\infty}{2\pi} \bar{\gamma}(1, y) \ln(1 - x) - \frac{ik}{2\pi} \bar{\Gamma} \ln(1 - x) + 0(1) \quad (6)$$

we see that this is only possible if

$$\bar{\gamma}(1, y) = -ik\bar{\Gamma}(y)/\lambda_\infty \quad (7)$$

This integral equation-formulated Kutta condition only coincides with the pressure-formulated condition in the limit of infinite aspect ratio, in which case $\lambda_\infty \to 1$, the incompatibility arising as a consequence of the Laidlaw approximation (2). From (4) and (7) we see that we should choose $f(\pi, y) = -ik\bar{\Gamma}(y)/\lambda_\infty$, but we should not choose $f(\phi, y) = \text{const} = f(\pi, y)$, since this would introduce a $\ln(1 + x)$ singularity from the Cauchy term at the leading edge. To avoid this, we choose $f(0, y) = 0$ so that a suitable choice would be, for example,

$$f(\phi, y) = -ik\bar{\Gamma}(y)\phi/\pi\lambda_\infty \quad (8)$$

We see that Laidlaw's additional term does not remove the logarithmic downwash singularity except in the lifting line or two-dimensional limit and, in fact, introduces one at the leading edge. The properly modified series, (4) and (8), does, on the other hand, lead to a well-defined collocation problem.

In order to assess the numerical importance of the additional term, let us apply the modified and unmodified series to the two-dimensional problem of an airfoil performing vertical translation oscillations since the exact solution is well known[5] and affords a means of comparison. Collocating at $x = 0$, ± 0.4, ± 0.8, we find that, for $k = 0.5$ and 1.0,† the lift predicted using an unmodified series is in error by 12.4 and 19.2%, respectively, compared to 0.3 and 0.1% using the modified series given by (4) and (8) with $\lambda_\infty = 1$.

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† Reduced frequencies in excess of unity are typical in unsteady propeller theory,[6] for example.

Finally, we point out that the application of these results is not limited to unsteady three-dimensional airfoil theory as governed by (1) subject to the approximation (2). The essential feature is the presence of the singular wake integral (together with the absence of an exact solution). The two-dimensional unsteady, as well as the steady jet-flapped, hydrofoil near a free surface represent suitable cases.

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An Analytical Approach to Hypervelocity Impact

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Nomenclature

C = sound velocity behind the shock wave given in Table III of Ref. 2
e = internal energy per unit mass
k = thermal conductivity
P = pressure
S = constant defined in Table III of Ref. 2
T = temperature
u = particle velocity
x = distance traveled by the projectile during penetration
μ = dynamic viscosity
ρ = mass density of the material

Introduction

In a previous paper by Yuan and Scully,[1] a simplified theory of penetration in hypervelocity impact was presented. This theory is based on the assumption that, during a very short interval of time, the energy released by the impact is sufficient to melt or to vaporize both the projectile and the small volume of target involved. In this regime of fluid impact, a transmitted shock is known to propagate into the target and a reflected shock propagates into the projectile. The motion of the fluid which represents the shock front is assumed to be governed by the basic equations of one-dimensional flow of a viscous compressible fluid. A solution in closed form was obtained from these equations which gives the penetration parameter as a function of the impact velocity and the sonic velocity of the material at standard conditions.

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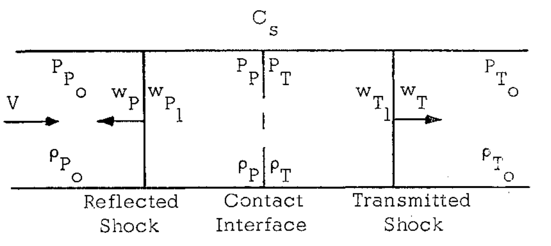

Fig. 1 Shock wave of the impact model.

Although the results of this penetration and impact velocity relation agree quite well with available experimental data for alike projectile and target materials, the equation of state for a perfect gas was used in the analysis.

In the present paper, a more realistic equation of state is introduced to replace the equation of state for a perfect gas in the basic equations for a viscous compressible fluid. This equation of state was determined through shock-wave velocity and free-surface velocity measurements. These measurements were obtained from experiments of a plane-wave explosive system in which a thin plate, moving at several kilometers per second, was used to produce strong shock waves in a stationary target plate.[2] A theoretical solution is obtained from the one-dimensional equations of a viscous compressible fluid. The results give a penetration parameter as a function of an impact velocity parameter. The constants in the theoretical expression are determined from the experimental data for identical projectile and target materials.

Based on the experimental data for three different materials, the present expression for the penetration-impact velocity relation can be simplified after dropping small terms. The result is identical to the expression for the penetration-impact velocity relation given in the previous paper.[1] Hence, it is evident that the results obtained in Ref. 1 by using the equation of state for a perfect gas are believed to be justified.

Hypervelocity Impact Model

When the stresses of the target material, caused by impact, are much greater than the yield stress, it is believed that both the projectile and target materials will eventually become fluid. As the velocity of impact approaches the same order of magnitude as the velocity of dilatational waves in the target material, shock waves are generated in the target. A conventional one-dimensional shock-wave propagation is shown in Fig. 1 to illustrate the type of impact under consideration.[3, 4]

When the projectile strikes the target at velocity V, a transmitted shock propagates into the target at velocity w_T and a reflected shock propagates into the projectile at velocity w_p. Since there is no penetration mixing of the projectile and target materials, the contact interface is established in the ideal mechanical impact model. The contact surface C_s is then separating projectile and target materials and is traveling in the direction of the transmitted shock at the particle velocity.

If the motion of the particles behind the shock front is considered steady and the shear stress is neglected, then the relation between the particle velocity behind the shock front u and the impact velocity V for alike projectile and target materials is given by[3]

$$u = V/2 \tag{1}$$

Fundamental Equations

When both the projectile and target are assumed to be in a fluid state under the high-speed impact process, the motion of the fluid which represents the shock front is governed by the following equations for one-dimensional flow of a viscous compressible fluid without a body force:

$$P = \frac{C^2\rho[(\rho/\rho_0) - 1]}{\{(\rho/\rho_0) - S[(\rho/\rho_0) - 1]\}^2} \tag{2}$$